Regression Analysis Using GIS

By Jennie Murack
murack@mit.edu

Regression Basics

How is Regression Different from other Spatial Statistical Analyses?

- With other tools you ask **WHERE** something is happening?

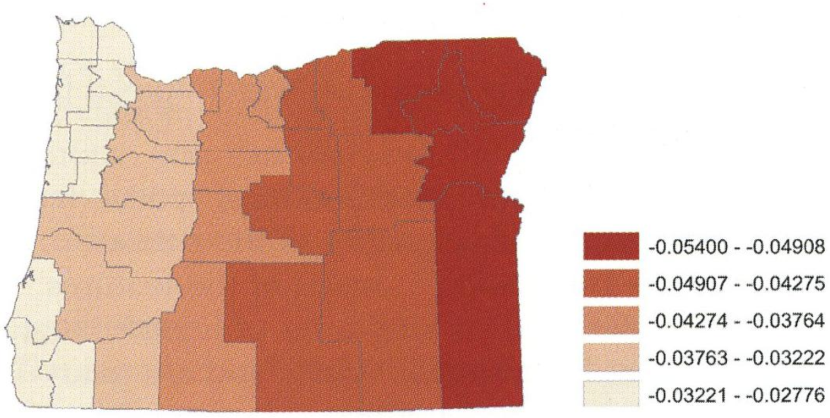
  - Are there places in the United States where people are persistently dying young?
  - Where are the hot spots for crime, 911 emergency calls, or fires?
  - Where do we find a higher than expected proportion of traffic accidents in a city?

With Regression Analyses, you ask **WHY** something is happening.

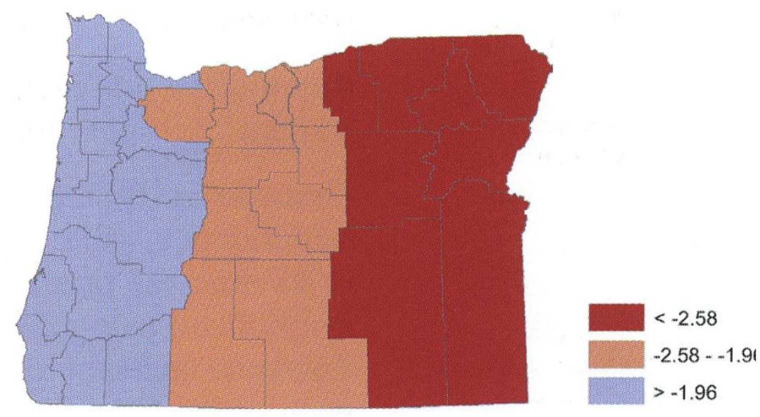
- Why are there places in the United States where people persistently die young? What might be causing this?
- Can we model the characteristics of places that experience a lot of crime, 911 calls, or fire events to help reduce these incidents?
- What are the factors contributing to higher than expected traffic accidents? Are there policy implications or mitigating actions that might reduce traffic accidents across the city and/or in particular high accident areas?

Regression analysis allows you to…

- Model, examine, and explore spatial relationships
- Predict



Coefficients for percent rural and low-weight births



T-scores show where this relationship is significant

Reasons to Use Regression Analysis

- To model phenomenon in order to better understand it and possibly make decisions
- To model phenomenon to predict values at other places or times
- To explore hypotheses

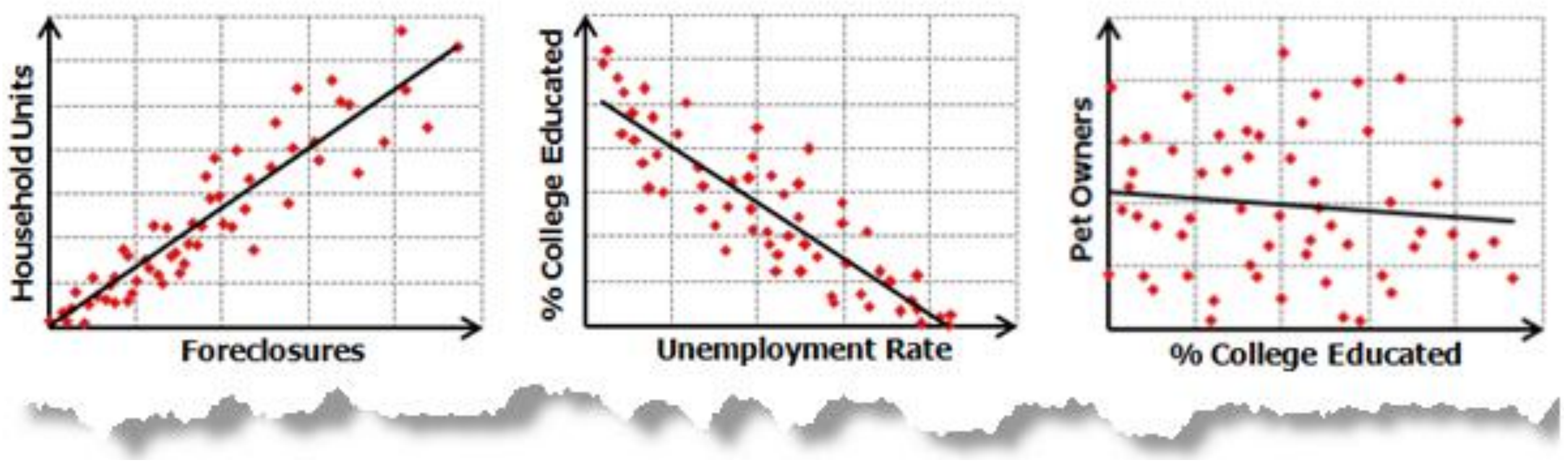
Regression Models

Spatial Regression

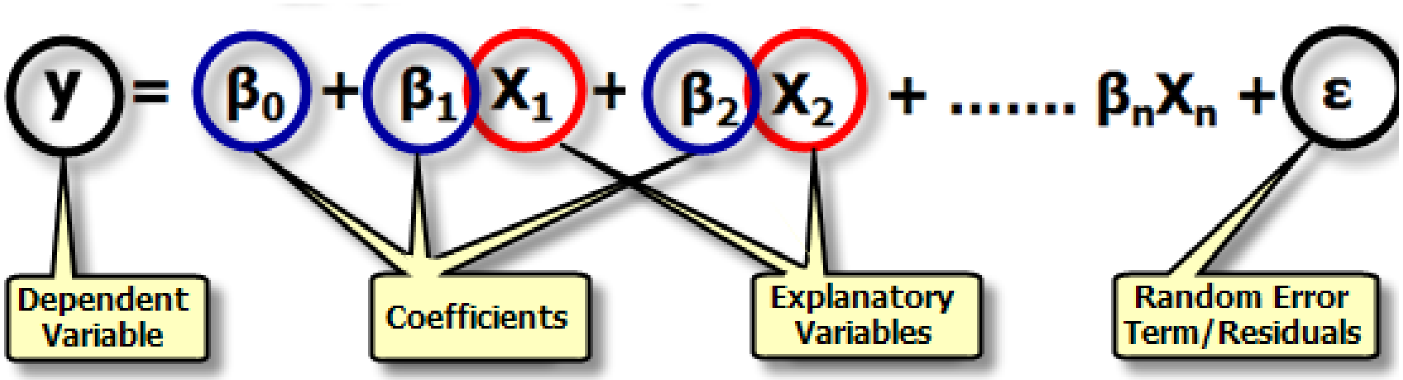
- Spatial data often do not fit traditional, non-spatial regression requirements because they are:
  - spatially autocorrelated (features near each other are more similar than those further away)
  - nonstationary (features behave differently based on their location/regional variation)
- No spatial regression method is effective for both characteristics.

Linear Regression

- Used to analyze linear relationships among variables.
- Linear relationships are positive or negative
- Regression analyses attempt to demonstrate the degree to which one or more variables potentially promote positive or negative change in another variable.



Linear Regression Equation



Y = variable you are trying to predict or understand

X = value of the dependent variables

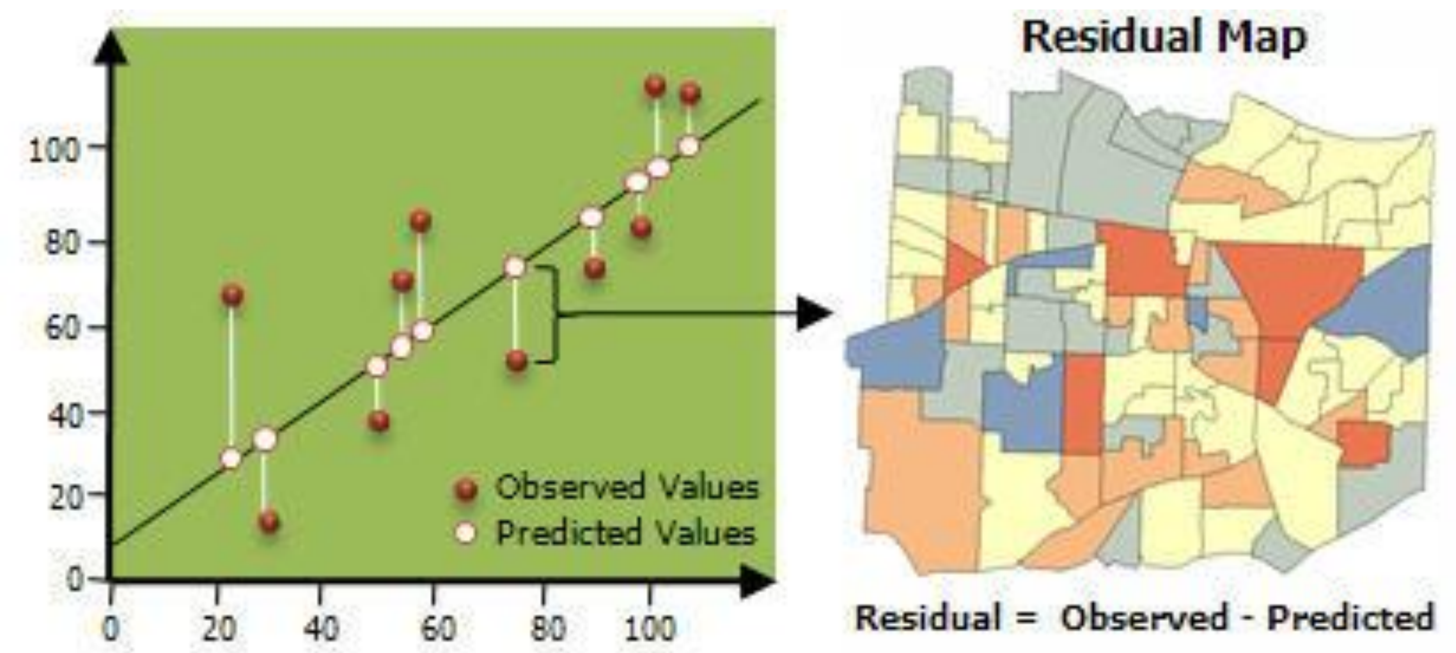
β = coefficients computed by the regression tool, represent the strength and type of relationship X has to Y

Residuals = the unexplained portion of the dependent variable

- large residuals = a poor model fit

Residuals

Difference between the observed and predicted values



Ordinary Least Squares Regression

- Best known technique and a good starting point for all spatial regression analyses.
- Global model = provides 1 equation to represent the entire dataset
- Available in Geoda and ArcMap

Geographically Weighted Regression (GWR) (ArcMap)

- Provides a local model of the variable by fitting a regression equation to every feature in the dataset.
- The equations incorporate the dependent and explanatory variables of features falling within the bandwidth of each target feature.

Spatial Lag Model (Geoda)

- Includes a spatially lagged dependent variable: $y=(\rho)Wy + X(\beta) + \varepsilon$
  - Wy = spatially lagged dependent variable for weights matrix W
  - X = matrix of observations on the explanatory variable
  - ε = vector of error terms
  - ρ and β are parameters
- A spatial lag is a variable that averages the neighboring values of a location.
- Accounts for autocorrelation in the model with the weights matrix
  - y is dependent on its neighbors (through the weights matrix)

Spatial Error Model (Geoda)

- Includes a spatial autoregressive error term: $y=X(\beta) + \varepsilon$, $\varepsilon = \lambda(W)\,\varepsilon + u$
  - W is the spatial weights matrix
  - X is a matrix of observations on the explanatory variables
  - ε is a vector of spatially autocorrelated error terms
  - u is a vector of i.i.d. (independent identically distributed) errors
  - λ and β are parameters.
- Similar to spatial lag model: accounts for autocorrelation in the error with the weights matrix.

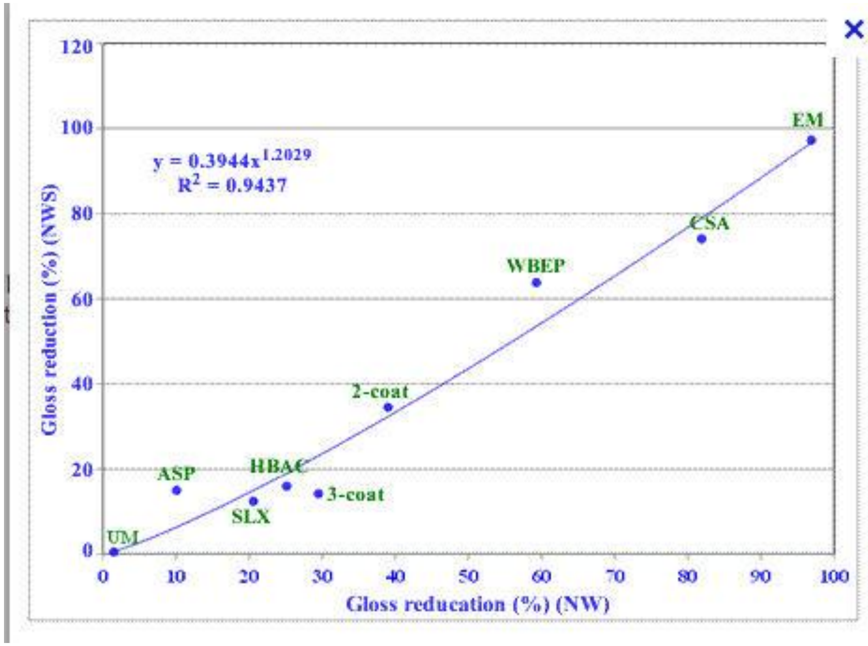
Interpreting Results

Summary Statistics

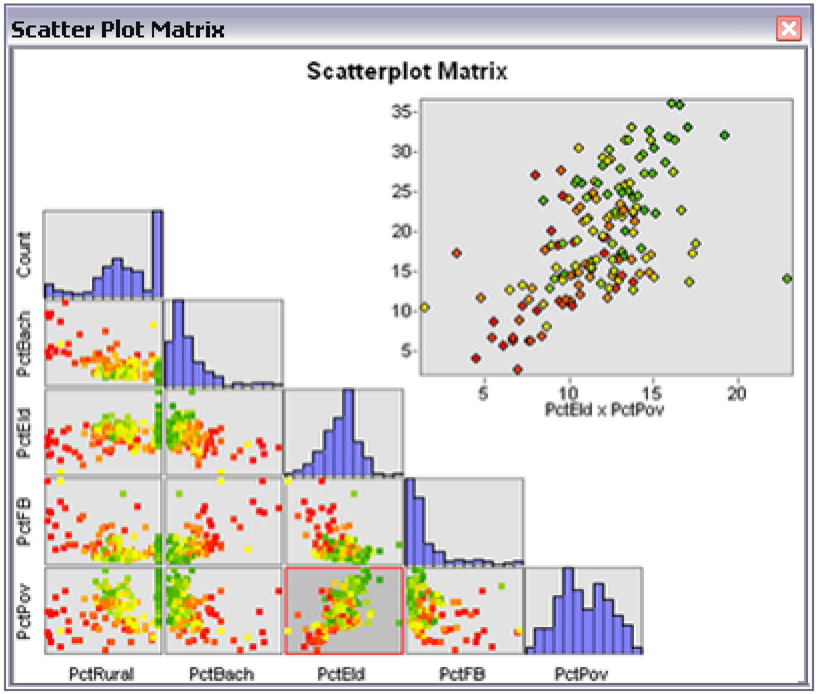
- Mean/Standard Deviation
- Number of observations
- Dependent Variable

Measure of Regression Fit

- R^2
  - How well the regression line fits the data
  - The proportion of variability in the dataset that is accounted for by the regression equation.
  - Ranges from 0 to 1
  - Outliers or non-linear data could decrease R^2.



Data Outliers



- Create a scatter plot to examine extreme values and correct or remove outliers if possible.

Solutions:

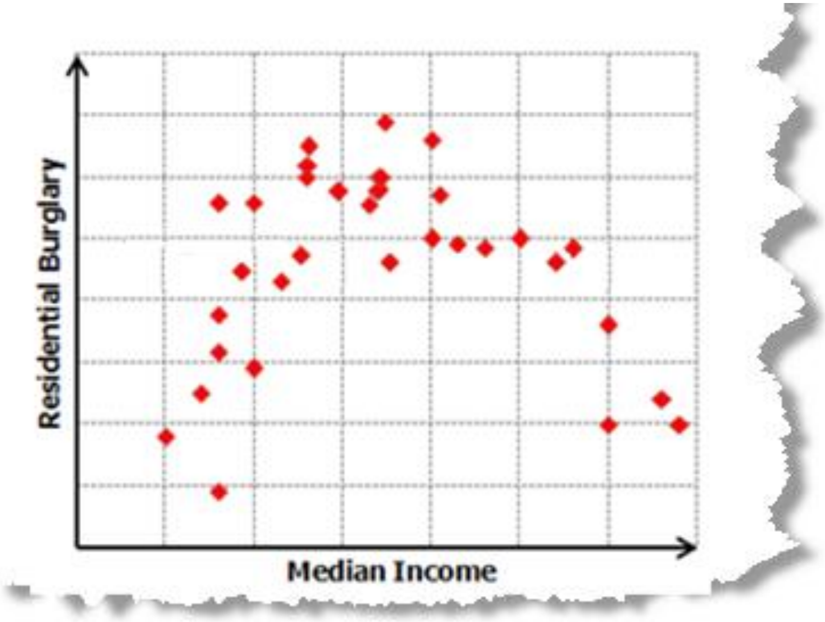
- Run regression with and without outliers to see their effect on the analysis

Nonlinear Relationships

Solutions:

- Create a scatter plot matrix graph and transform variables

- Use a non-linear regression model



Variable Coefficients

- The sign shows whether the relationship is positive or negative
- The coefficient shows the strength of the relationship.
- P-values indicate whether the variable is a significant predictor of the independent variable.
- Use the coefficients to form a regression equation: $y = 10 + .5a – 6b + 8c$
- Remove variables with high p-values to see if R^2 increases.

Comparability

- Use Akaike information Criterion (AICc) value when comparing models.
- AICc is a measure of the relative goodness of fit of a statistical model.
- It assists with model selection, but does not test the null hypothesis.
- A lower AICc value means the model is a better fit for the data.

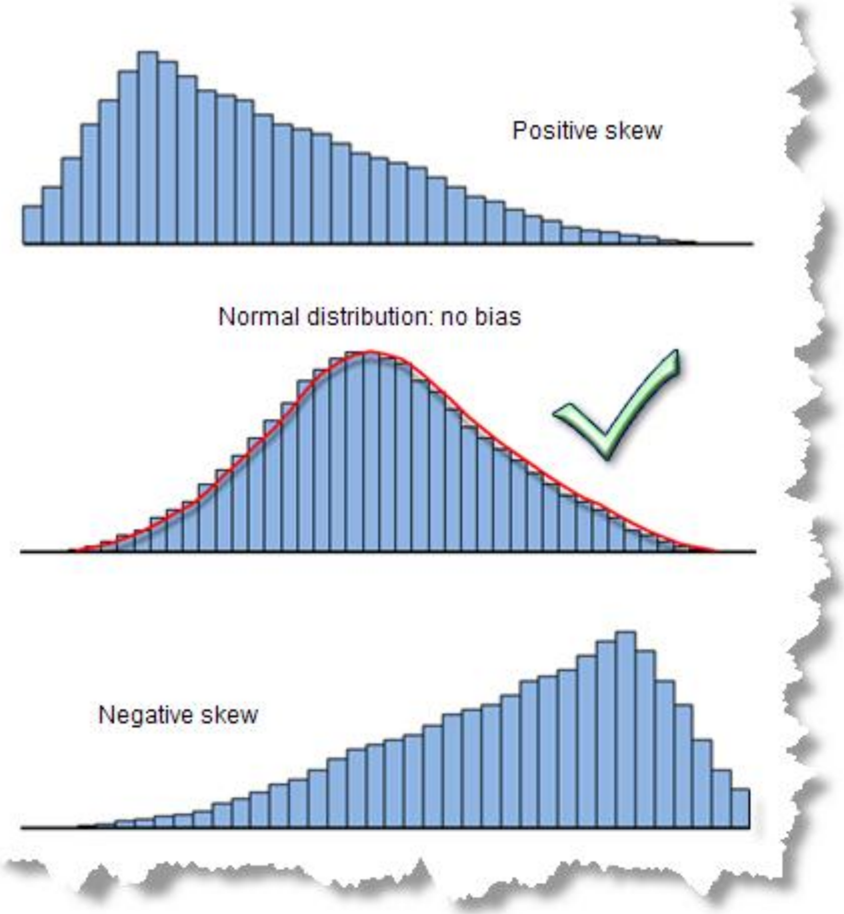
Multicollinearity

- Two or more variables may be highly correlated with one another
- Variance Inflation Factor (VIF)
  - Larger than 7.5 could indicate redundancy among variables.
- Multicollinearity Condition Number
  - Values over 30 indicate a problem

Tests for Residuals/Errors

- **Jarque-Bera Test**: Tests the normality of errors. If it is significant, you may be missing an explanatory variable.
- **Breusch-Pagan**, **Koenker-Bassett**, **White**: Test for heteroskedasticity (non-constant variance). If these are significant, the relationships between some or all of the explanatory variables and the dependent variable are non-stationary (a strong predictor in one area, but weak in others). Try other regression models (GWR, etc.)
- **Spatial Autocorrelation**: Autocorrelated residuals could indicate missing variables or the need for alternative regression models.

Normal Distribution Bias



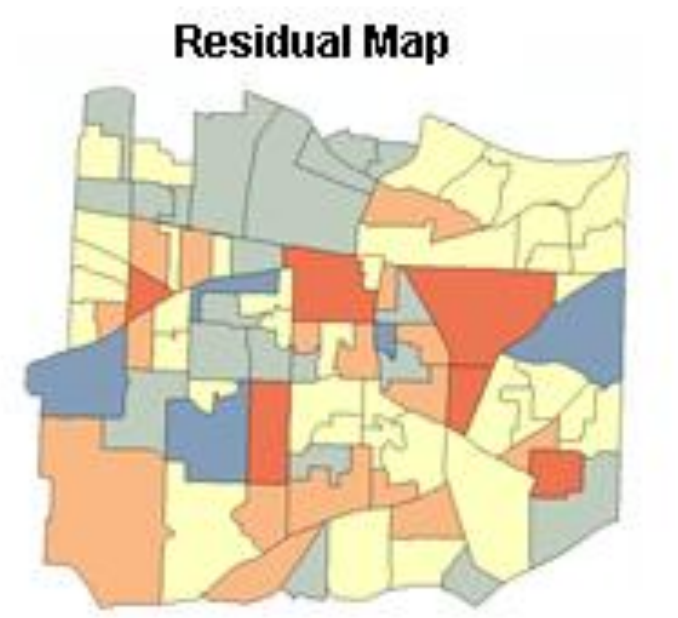
Solutions:

- Jarque-Bera tests whether residuals are normally distributed.
- Model may be misspecified or nonlinear.

Spatially autocorrelated residuals

Solutions:

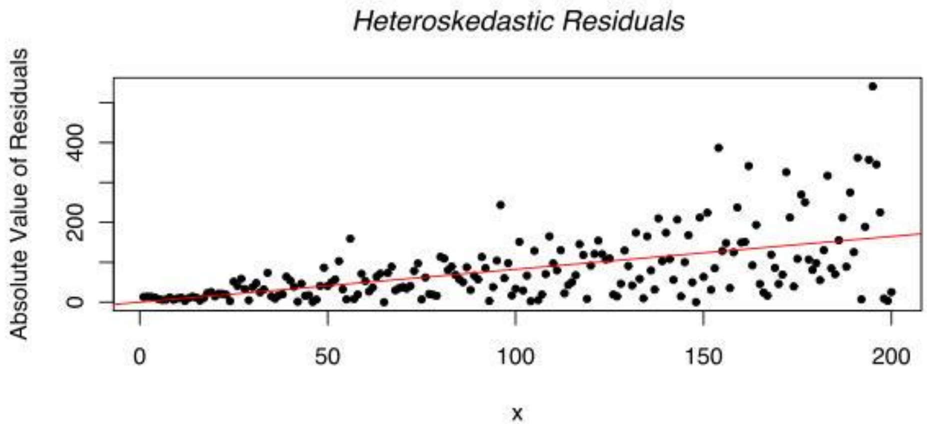
- Run the spatial autocorrelation tool on the residuals.
- If there is significant clustering, there could be misspecification (a variable is missing from the model).

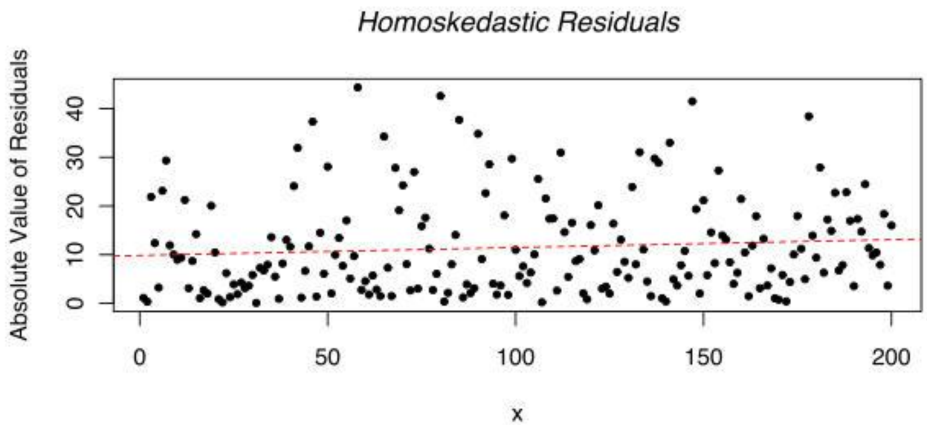


Plotting Residuals

- Residuals vs. ID (or any unique identifier)
  - Should not display any pattern
  - Examine large residuals and look for systematic relationships to improve upon the model
- Residuals vs. Predicted
  - Detects heteroskedasticity, or unequal variances
  - Funnel-like patterns indicate relationships between the residuals and predicted values

Heteroskedasticity





- If heteroskedasticity exists, variability differs across sub-populations.

- Variables could be strong predictors in some areas, but weak predictors in others.

Maps

- Predicted Value Map
  - The value of the dependent variable, based on the regression equation
  - A smoothed map
  - Random variability, due to factors other than those included in the model, have been smoothed out
- Residual Map
  - Indicates systematic over or under prediction in regions, which could be evidence of spatial autocorrelation

Summary

Steps of Regression

- Determine what you are trying to predict or examine (dependent variable)
- Identify key explanatory variables
- Examine the distribution to determine the type of regression to conduct
- Run the regression
- Examine the coefficients
- Examine the residuals
  - The mean should equal 0.
  - They should create a random pattern.
  - They should create a normal distribution.
  - Problems could indicate missing variables.
- Remove or add variables and repeat regression
- Use another regression model if necessary.

Resources

Regression Resources

- ESRI Spatial Statistics Website:
  http://blogs.esri.com/Dev/blogs/geoprocessing/archive/2010/07/13/Spatial-Statistics-Resources.aspx
- Geoda Workbook:
  https://geodacenter.asu.edu/system/files/geodaworkbook.pdf
- ESRI Regression Tool Help:
  http://resources.arcgis.com/en/help/main/10.1/index.html#/An_overview_of_the_Modeling_Spatial_Relationships_toolset/005p0000001w000000/
- Video lecture on Spatial Lag and Error:
  https://geodacenter.asu.edu/spatial-lag-and

Survey

- Access our workshop survey at the following site: http://libguides.mit.edu/gisworkshops

- Click on Survey in the Regression box on the left side of the page.